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2

Decision Making

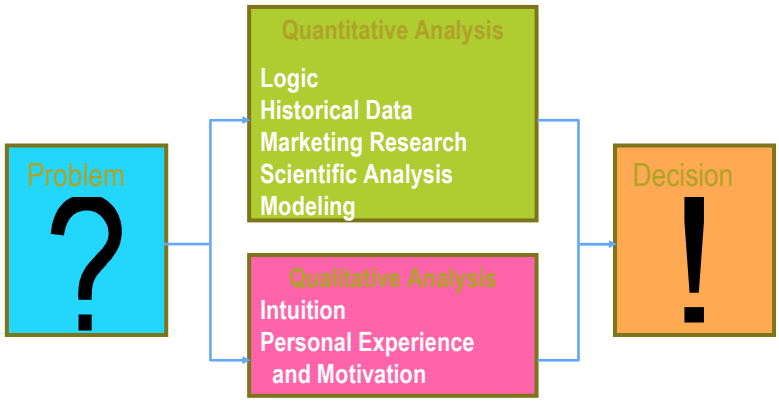
Basic techniques

Limitations of Models

They

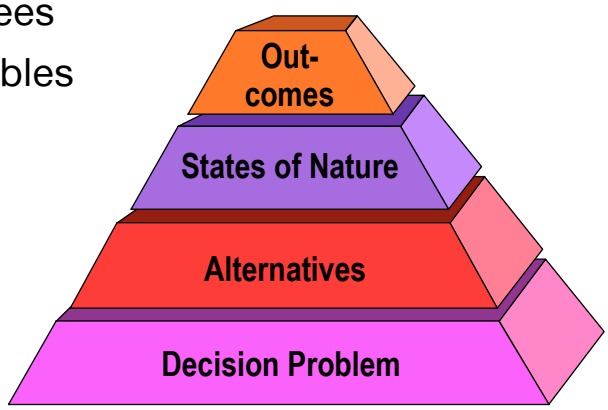
- may be expensive and time-consuming to develop and test
- are often misused and misunderstood (and feared) because of their mathematical and logical complexity
- tend to downplay the role and value of nonquantifiable information
- often have assumptions that oversimplify the variables of the real world

The Decision-Making Process



Ways of Displaying a Decision Problem

- Decision trees
- Decision tables



Fundamentals of Decision Theory

The three types of decision models:

- Decision making under uncertainty
- Decision making under risk
- Decision making under certainty

Fundamentals of Decision Theory - continued

Terms:

- Alternative: course of action or choice
- State of nature: an occurrence over which the decision maker has no control

Symbols used in decision tree:

- □ A decision node from which one of several alternatives may be selected
- ○ A state of nature node out of which one state of nature will occur

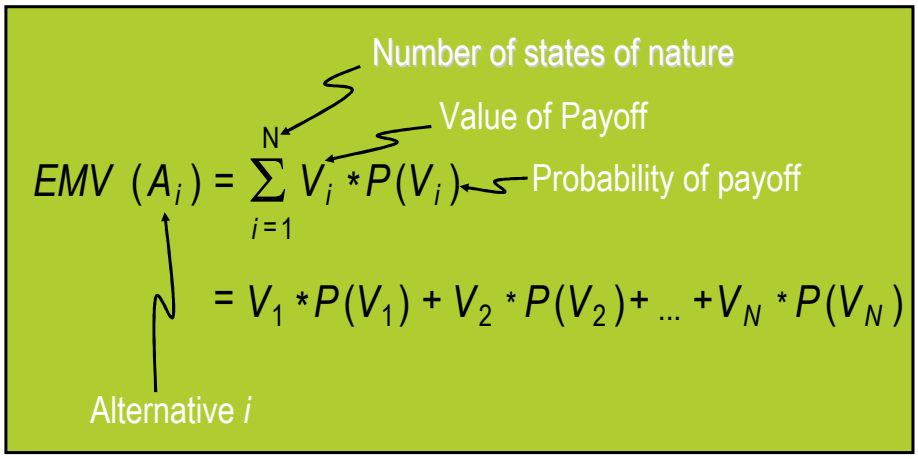
Decision Table

Alternatives	States of Nature	
	State 1	State 2
Alternative 1	Outcome 1	Outcome 2
Alternative 2	Outcome 3	Outcome 4

Decision Making Under Risk

- Probabilistic decision situation
- States of nature have probabilities of occurrence
- Select alternative with largest expected monetary value (EMV)
  - EMV = Average return for alternative if decision were repeated many times

Expected Monetary Value Equation



Decision Making Under Uncertainty

- Maximax - Choose the alternative that maximizes the maximum outcome for every alternative (Optimistic criterion)
- Maximin - Choose the alternative that maximizes the minimum outcome for every alternative (Pessimistic criterion)
- Equally likely - chose the alternative with the highest average outcome.

Example - Decision Making Under Uncertainty

	States of Nature				
Alternatives	Favorable Market	Unfavorable Market	Maximum in Row	Minimum in Row	Row Average
Construct large plant	$200,000	-$180,000	$200,000	-$180,000	$10,000
Construct small plant	$100,000	-$20,000	$100,000	-$20,000	$40,000
	$0	$0	$0	$0	$0

Maximax Maximin Equally likely

Example - Decision Making Under Uncertainty

	States of Nature		
Alternatives	Favorable Market P(0.5)	Unfavorable Market P(0.5)	Expected value
Construct large plant	$200,000	-$180,000	$10,000
Construct small plant	$100,000	-$20,000	$40,000
Do nothing	$0	$0	$0

Best choice

Decision under uncertainty

- Laplace Criterion
- Minimax Criterion
- Savage (Regret) Criterion
- Hurwicz Criterion

Probabilities	0	0	0	0		
Option 1	0	3	10	10	0	10
Option 2	3	0	0	8	0	8
Option 3	16	11	4	6	4	16
Option 4	25	15	11	0	0	25
			minimum		0	8
					minimin	minimax

Probabilities	0	0	0	0	
Option 1	0	3	10	10	10
Option 2	3	0	0	8	8
Option 3	16	11	4	6	16
Option 4	25	15	11	0	25
Minimax regret					8

Probabilities	0	0	0	0			
Option 1	0	3	10	10	0	10	5
Option 2	3	0	0	8	0	8	4
Option 3	16	11	4	6	4	16	10
Option 4	25	15	11	0	0	25	12.5
				maximum	4	25	12.5
					maximin	maximax	Best Hurwicz

Alfa=.5

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Extreme analysis

Expected Value of Perfect Information (*EVPI*)

- EVPI places an upper bound on what one would pay for additional information
- EVPI is the expected value with perfect information minus the maximum EMV

Expected Value With Perfect Information (*EV|PI*)

$$EV|PI = \sum_{j=1}^{n} (\text{Best outcome for the state of nature } j) * P(S_j)$$

where j=1 to the number of states of nature, *n*

Expected Value of Perfect Information

- EVPI = EV|PI - maximum EMV

Expected Value of Perfect Information

	State of Nature		
Alternative	Favorable Market ($)	Unfavorable Market ($)	EMV
Construct a large plant	200,000	-$180,000	$20,000
Construct a small plant	$100,000	$20,000	$40,000
Do nothing	$0	$0	$0
Probabilities	0.50	0.50	

Expected Value of Perfect Information

EVPI = expected value with perfect information - max(EMV)

= $200,000*0.50 + 0*0.50 - $40,000

= $60,000

Expected Opportunity Loss

- EOL is the cost of not picking the best solution
- EOL = Expected Regret

Computing EOL - The Opportunity Loss Table

	State of Nature	
Alternative	Favorable Market ($)	Unfavorable Market ($)
Large Plant	200,000 - 200,000	0 - (-180,000)
Small Plant	200,000 - 100,000	0 -(-20,000)
Do Nothing	200,000 - 0	0-0
Probabilities	0.50	0.50

The Opportunity Loss Table - continued

	State of Nature	
Alternative	Favorable Market ($)	Unfavorable Market ($)
Large Plant	0	180,000
Small Plant	100,000	20,000
Do Nothing	200,000	0
Probabilities	0.50	0.50

The Opportunity Loss Table - continued

Alternative		**EOL**
Large Plant	**(0.50)*$0 + (0.50)*($180,000)**	**$90,000**
Small Plant	**(0.50)*($100,000) + (0.50)(*$20,000)**	**$60,000**
Do Nothing	**(0.50)*($200,000) + (0.50)*($0)**	**$100,000**

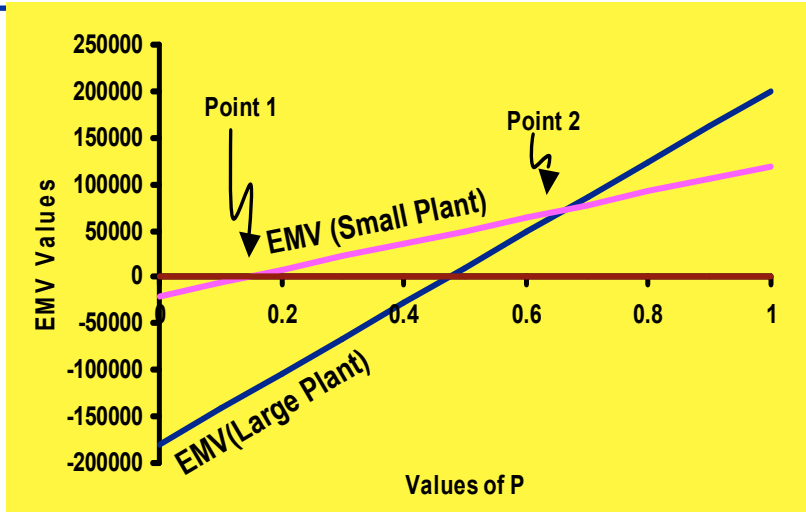
Sensitivity Analysis

EMV(Large Plant) = $200,000P - (1-P)$180,000

EMV(Small Plant) = $100,000P - $20,000(1-P)

EMV(Do Nothing) = $0P + 0(1-P)

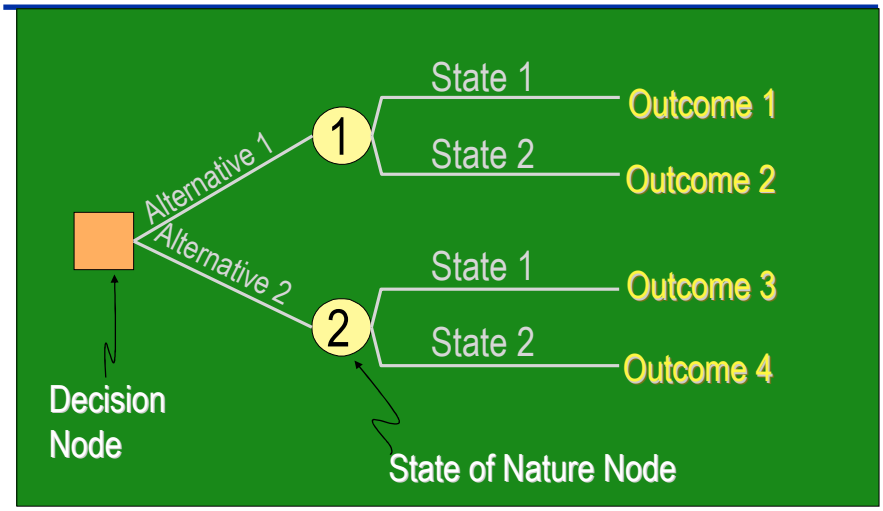
Sensitivity Analysis - continued



Decision Trees

- Graphical display of decision process
- Used for solving problems
  - With 1 set of alternatives and states of nature, decision tables can be used also
  - With several sets of alternatives and states of nature (sequential decisions), decision tables cannot be used
- EMV is criterion most often used

Decision Tree



Analyzing Problems with Decision Trees

- Define the problem
- Structure or draw the decision tree
- Assign probabilities to the states of nature
- Estimate payoffs for each possible combination of alternatives and states of nature
- Solve the problem by computing expected monetary values for each state-of-nature node

هُوَالْحَقّ

Game theory

Basic payoff tables

مثال – تئوری بازیها (استراتژی شیر یا خط)

	Col strat1	Col strat2	Row Minimum	Maximin
Row strategy 1	1.	-1.	-1.	-1.
Row strategy 2	-1.	1.	-1.	-1.
Column Maximum	1.	1.		
Minimax	1.	1.		
-1 <= value <= 1				

مثال – بازی دو نفره دارای نقطه زینی Saddle point

Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
1	8.	2.	9.	5.	0.
2	6.	5.	7.	18.	1.
3	7.	3.	-4.	10.	0.
Column Mix--->	0.	1.	0.	0.	
Value of game (to row)	5.				

Maximin & Minimax

<untitled>

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
1	8.	2.	9.	5.	2.	
2	6.	5.	7.	18.	5.	5.
3	7.	3.	-4.	10.	-4.	
Column Maximum	8.	5.	9.	18.		
Minimax		5.				
Value= 5						

مسئله بازی دو نفره – عدم تعادل

Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	5.	-10.	9.	0.	0.
Row strategy 2	6.	7.	8.	1.	0.
Row strategy 3	8.	7.	15.	2.	0.2778
Row strategy 4	3.	4.	-1.	4.	0.7222
Column Mix--->	0.	0.	0.1111	0.8889	
Value of game (to row)	3.4444				

Maximin & Minimax

<untitled>

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	5.	-10.	9.	0.	-10.	
Row strategy 2	6.	7.	8.	1.	1.	
Row strategy 3	8.	7.	15.	2.	2.	2.
Row strategy 4	3.	4.	-1.	4.	-1.	
Column Maximum	8.	7.	15.	4.		
Minimax				4.		
2 <= value <= 4						

Payoff profit matrix

	s1	s2	s3	s4	s5
A1	15	10	0	-6	17
A2	3	14	8	9	2
A3	1	5	14	20	-3
A4	7	19	10	2	0

Compare the solutions

- Laplace Criterion
- Minimax Criterion
- Savage (Regret) Criterion
- Hurwicz Criterion (alfa=.5)

- Find saddle point
- Value of the game

	B			
A	8	6	2	8
	8	9	4	5
	7	5	3	5

- Find saddle point
- Value of the game

	B			
A	4	-4	-5	6
	-3	-4	-9	-2
	6	7	-8	-9
	7	3	-9	5

- Find a range for P & Q for (2,2) saddle point game

	B			
A	1	P	3	
	Q	5	10	
	6	2	3	

Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	8.	6.	2.	8.	0.
Row strategy 2	8.	9.	4.	5.	1.
Row strategy 3	7.	5.	3.	5.	0.
Column Mix--->	0.	0.	1.	0.	
Value of game (to row)	4.				

<untitled>

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Row strategy 1	8.	6.	2.	8.	2.	
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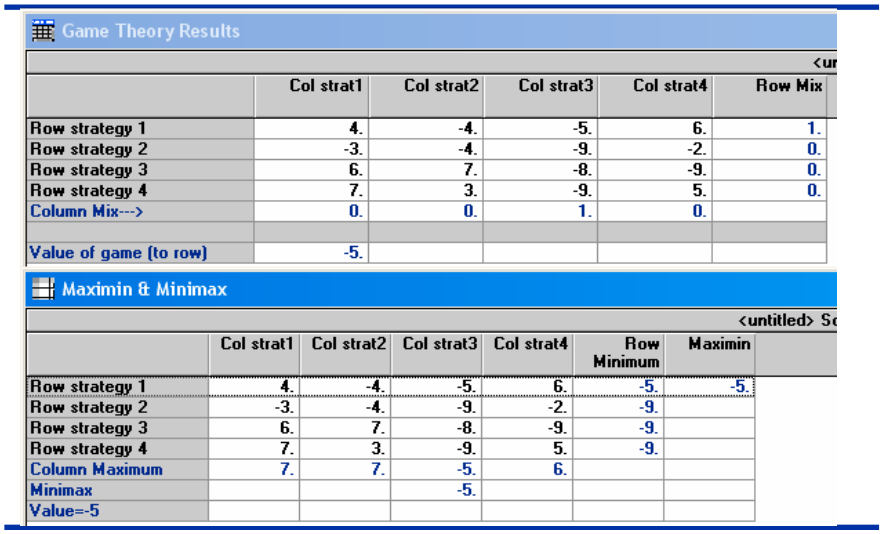
Column Maximum	8.	9.	4.	8.		
Minimax			4.			
Value= 4						



Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	4.	-4.	-5.	6.	1.
Row strategy 2	-3.	-4.	-9.	-2.	0.
Row strategy 3	6.	7.	-8.	-9.	0.
Row strategy 4	7.	3.	-9.	5.	0.
Column Mix--->	0.	0.	1.	0.	
Value of game (to row)	-5.				

Maximin & Minimax

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	4.	-4.	-5.	6.	-5.	-5.
Row strategy 2	-3.	-4.	-9.	-2.	-9.	
Row strategy 3	6.	7.	-8.	-9.	-9.	
Row strategy 4	7.	3.	-9.	5.	-9.	
Column Maximum	7.	7.	-5.	6.		
Minimax			-5.			
Value=-5						

Jamshid Nazemi *Multi-Criteria Decision Making* Slide no.: 41

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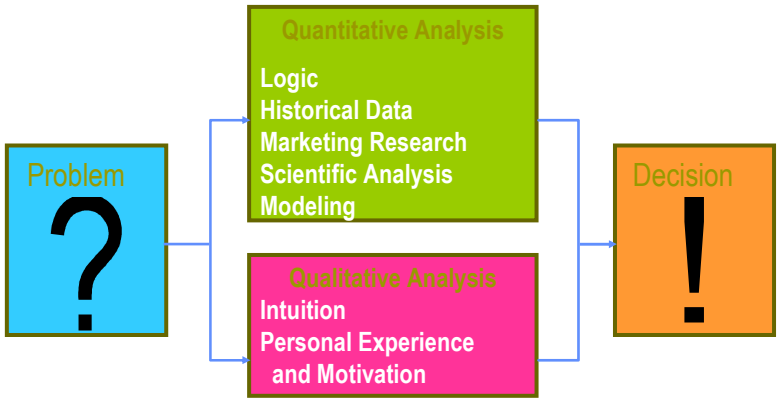

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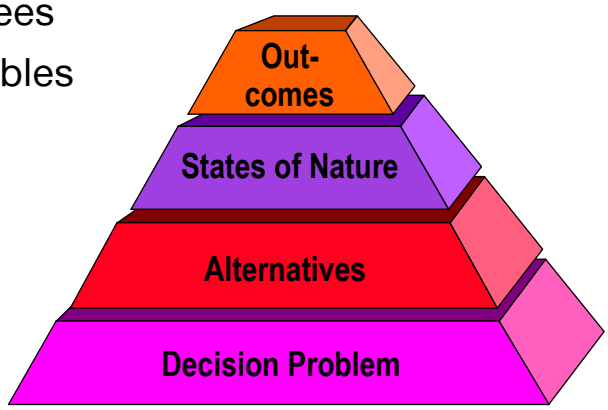
- Decision tables



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The three types of decision models:

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Decision Table

Alternatives	States of Nature	
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Decision Making Under Risk

- Probabilistic decision situation
- States of nature have probabilities of occurrence
- Select alternative with largest expected monetary value (EMV)
  - EMV = Average return for alternative if decision were repeated many times

Expected Monetary Value Equation

Number of states of nature

Value of Payoff

$$EMV\ (A_i) = \sum_{i=1}^{N} V_i * P(V_i)$$

Probability of payoff

$$= V_1 * P(V_1) + V_2 * P(V_2) + \ldots + V_N * P(V_N)$$

Alternative *i*

Decision Making Under Uncertainty

- Maximax - Choose the alternative that maximizes the maximum outcome for every alternative (Optimistic criterion)
- Maximin - Choose the alternative that maximizes the minimum outcome for every alternative (Pessimistic criterion)
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	$0	$0	$0	$0	$0

Maximax Maximin Equally likely

Example - Decision Making Under Uncertainty

	States of Nature		
Alternatives	Favorable Market P(0.5)	Unfavorable Market P(0.5)	Expected value
Construct large plant	$200,000	-$180,000	$10,000
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Do nothing	$0	$0	$0

Best choice

Decision under uncertainty

- Laplace Criterion
- Minimax Criterion
- Savage（Regret）Criterion
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Probabilities	0	0	0	0		
Option 1	0	3	10	10	0	10
Option 2	3	0	0	8	0	8
Option 3	16	11	4	6	4	16
Option 4	25	15	11	0	0	25
			minimum		0	8
					minimin	minimax

Probabilities	0	0	0	0	
Option 1	0	3	10	10	10
Option 2	3	0	0	8	8
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Minimax regret					8

Probabilities	0	0	0	0			
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					maximin	maximax	Best Hurwicz

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Extreme analysis

Expected Value of Perfect Information (*EVPI*)

- EVPI places an upper bound on what one would pay for additional information
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$$EV|PI = \sum_{j=1}^{n} (\text{Best outcome for the state of nature } j) * P(S_j)$$

where j=1 to the number of states of nature, *n*

Expected Value of Perfect Information

- EVPI = EV|PI - maximum EMV

Expected Value of Perfect Information

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Do nothing	$0	$0	$0
Probabilities	0.50	0.50	

Expected Value of Perfect Information

EVPI = expected value with perfect information - max(EMV)

= $200,000*0.50 + 0*0.50 - $40,000

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Expected Opportunity Loss

- EOL is the cost of not picking the best solution
- EOL = Expected Regret

Computing EOL - The Opportunity Loss Table

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Large Plant	**(0.50)*$0 + (0.50)*($180,000)**	**$90,000**
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Sensitivity Analysis

EMV(Large Plant) = $200,000P - (1-P)$180,000

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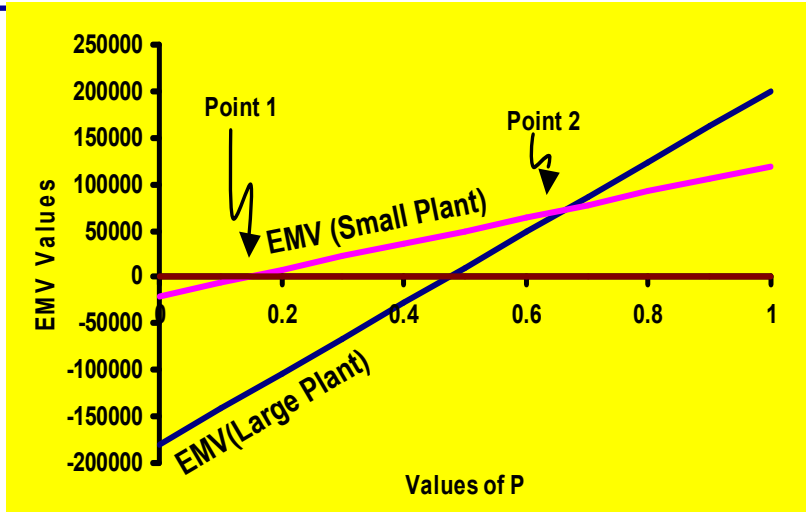
Sensitivity Analysis - continued



Decision Trees

- Graphical display of decision process
- Used for solving problems
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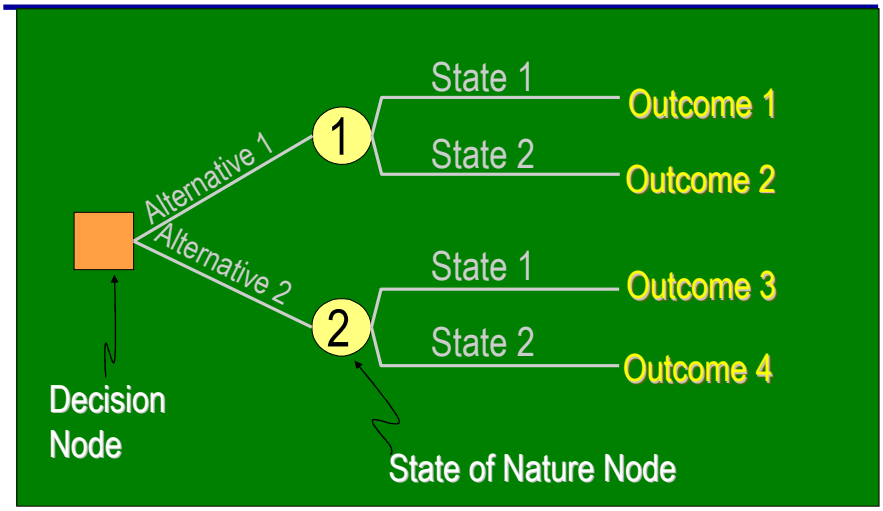
- EMV is criterion most often used

Decision Tree



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Row strategy 2	-1.	1.	-1.	-1.
Column Maximum	1.	1.		
Minimax	1.	1.		
-1 <= value <= 1				

مثال – بازی دو نفره دارای نقطه زینی Saddle point

Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
1	8.	2.	9.	5.	0.
2	6.	5.	7.	18.	1.
3	7.	3.	-4.	10.	0.
Column Mix--->	0.	1.	0.	0.	
Value of game (to row)	5.				

Maximin & Minimax

<untitled>

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
1	8.	2.	9.	5.	2.	
2	6.	5.	7.	18.	5.	5.
3	7.	3.	-4.	10.	-4.	
Column Maximum	8.	5.	9.	18.		
Minimax		5.				
Value= 5						

مسئله بازی دو نفره – عدم تعادل

Game Theory Results

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	5.	-10.	9.	0.	0.
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Value of game (to row)	3.4444				

Maximin & Minimax

<untitled>

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
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Row strategy 2	6.	7.	8.	1.	1.	
Row strategy 3	8.	7.	15.	2.	2.	2.
Row strategy 4	3.	4.	-1.	4.	-1.	
Column Maximum	8.	7.	15.	4.		
Minimax				4.		
2 <= value <= 4						

Payoff profit matrix

	s1	s2	s3	s4	s5
A1	15	10	0	-6	17
A2	3	14	8	9	2
A3	1	5	14	20	-3
A4	7	19	10	2	0

Compare the solutions

- Laplace Criterion
- Minimax Criterion
- Savage (Regret) Criterion
- Hurwicz Criterion (alfa=.5)

- Find saddle point
- Value of the game

	B			
A	8	6	2	8
	8	9	4	5
	7	5	3	5

- Find saddle point
- Value of the game

	B			
A	4	-4	-5	6
	-3	-4	-9	-2
	6	7	-8	-9
	7	3	-9	5

- Find a range for P & Q for (2,2) saddle point game

	B			
A	1	P	3	
	Q	5	10	
	6	2	3	